Addendum to paper "Closed Spaces in Cosmology"[1]

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Abstract

A few corrections and comments are made upon a previously published paper, on the subject of cosmological models with compact spatial sections.

KEY WORDS: Topology of the universe; closed Thurston and Bianchi types; spinor structure

The paper referred to in the title was published six years ago. Because of growing interest in this field - see, for example, the review paper by Lachièze-Rey and Luminet[2], or, for a recent work, Levin et al. [3] - we find it relevant now to publish the present Adendum-Errata.

1) On page 203 of [1], a term is missing in equation (4). The correct expression is

$$Du^a = (\partial u^a/\partial x^c + \Gamma^a_{bc} u^b)dx^c + (\partial u^a/\partial z)dz$$

2) On p. 204, Table II, row 5: for type BVI(A), form ω^3 should be $e^{(A+1)x}dz$.

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3) Regarding Theorem 2.1, p. 204: "If a closed space M admits a BKS metric, then M is locally homogeneous with respect to this metric." In the development of the proof in [1], it was stated that $M = \tilde{M}/\Gamma$, where $\Gamma \subset G$, with G the corresponding BKS group. As proved by Koike et al. [4], this is not possible for type BVIII and all Bianchi types of class B. But the theorem remains valid, with Γ a subgroup of the full group of the metric, $\mathrm{Isom}(\tilde{M})$.

Koike et al.'s result and Theorem 2.1 imply that a class B space can only be compactified if its full (orientation preserving) isometry group has a dimension larger than three. This explains why there are no closed spaces of types BIV and BVI($A \neq 0, 1$): their full isometry groups are of dimension three, and so essentially coincide with their Bianchi groups.

4) On Table III, p. 206, make the following corrections:

a) for type BII, $K_2 = K_3 = +1/4$

b) for type BIV, $K_1 = -3/4$

c) for type BVI(A), $d\lambda^2 = dx^2 + e^{2(A-1)x}dy^2 + e^{2(A+1)x}dz^2$

5) On p. 216, line 8, end of proof of Theorem 3.3: where is "and hence in Σ," it should be "and hence in $\tilde{\Sigma}$."

6) On p. 216, paragraph beginning with "Elementary particle theorists ...": the sentence "This structure is not unique, but ... dimensions involved." should be replaced with "This structure is not unique, but we can chooose one of the two alternatives as part of the *definition* of spinors. Cf. [5]."

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References

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[5] Penrose, R., and Rindler, W. (1984) *Spinors and Space-Time* (Cambridge University Press, Cambridge), p. 51.